

Nonlinear Inequalities and Systems of Nonlinear Inequalities

Objectives

- 1) Graph a nonlinear inequality
- 2) Graph a system of nonlinear inequalities.

Exam 5 Review materials

Nonlinear Inequalities and Systems of Inequalities

- Objectives
- 1) Graph a nonlinear inequality
 - 2) Graph a system of nonlinear inequalities

Review from Math 45:

① Graph $\begin{cases} y < -2x + 5 \\ y \geq 3x - 4 \end{cases}$

$$y < -2x + 5$$

a line $\Rightarrow$ dotted $<$ or $>$

slope -2 , y -int 5

$y <$ shade downward ($y \leq$)

OR

Test a point with known location not on line
on line

e.g. $(0,0)$

$$0 < -2(0) + 5$$

$$0 < 5 \text{ true}$$

shade the side of $y = -2x + 5$ that contains $(0,0)$

$$y \geq 3x - 4$$

a line $\Rightarrow$ solid $\geq$ or $\leq$

slope 3 , y -int -4

$y >$ shade upward ($y \geq$ also)

OR

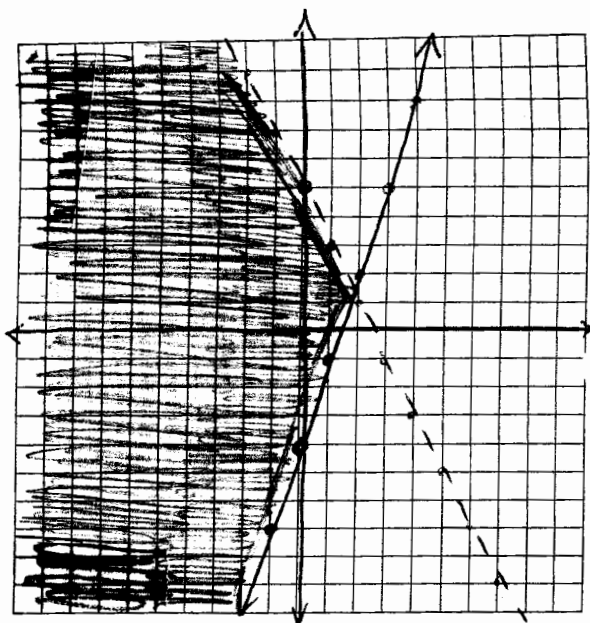
Test a point with known location not on the line

e.g. $(0,0)$

$$0 \geq 3(0) - 4$$

$$0 \geq -4 \text{ true}$$

Shade the side of $y = 3x - 4$ that contains $(0,0)$.



(If test point is false, shade the other side of the line, not containing the test point.)

- If the shaded areas do not overlap, the system of inequalities has no solution. (and no graph).

② Graph $\frac{x^2}{9} + \frac{y^2}{16} \geq 1$

step 1: Change $\geq$ to $=$, identify conic, and graph.

If $\geq$ or $\leq$, use a solid line
(If $<$ or $>$, use a dotted (dashed) line.)

$$\frac{x^2}{9} + \frac{y^2}{16} = 1 \text{ is an ellipse}$$

$$\frac{(x-0)^2}{9} + \frac{(y-0)^2}{16} = 1$$

center $(0,0)$

x-direction 3

y-direction 4

step 2: shade.

Method 1: Use a test point. For circles, ellipses and hyperbolas, use the center.

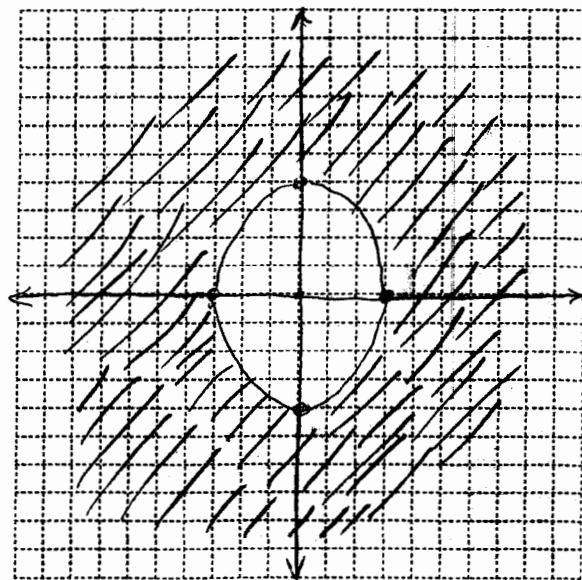
$(0,0)$

$$\frac{0^2}{9} + \frac{0^2}{16} \geq 1$$

$$0 + 0 \geq$$

$$0 \geq 1 \text{ false}$$

shade outside the ellipse



Method 2: For ellipses and circles

$$\left. \begin{array}{l} ax^2 + by^2 > \# \\ ax^2 + by^2 \geq \# \end{array} \right\}$$

greater than means outside

$$\left. \begin{array}{l} ax^2 + by^2 < \# \\ ax^2 + by^2 \leq \# \end{array} \right\}$$

less than means inside

To graph a system of nonlinear inequalities
shade only the OVERLAP of shaded areas

③ Graph $\begin{cases} 4y^2 > x^2 + 16 & (A) \\ y \geq x^2 - 3 & (B) \end{cases}$

Step 1: One inequality at a time,
very neatly.

$$4y^2 > x^2 + 16$$

> means dotted line; Arrange equation in standard form.

$$4y^2 > x^2 + 16$$

$$-16 > x^2 - 4y^2$$

$$x^2 - 4y^2 < -16$$

↑
subtract $\left. \begin{matrix} x^2 \text{ and } y^2 \end{matrix} \right\}$ hyperbola.

Need RHS 1.

$$\frac{x^2}{-16} - \frac{4y^2}{-16} > \frac{-16}{-16}$$

← Note: Exchanging LHS and RHS changes > to <
Note: Dividing by a negative changes < to >

$$\frac{-x^2}{16} + \frac{y^2}{4} > 1$$

← Negative at front of 1st term.
Exchange order of 1st & 2nd terms

$$\frac{y^2}{4} - \frac{x^2}{16} > 1$$

center (0,0) x-dir $\sqrt{16} = 4$ (under x^2)
y-dir $\sqrt{4} = 2$ (under y^2)

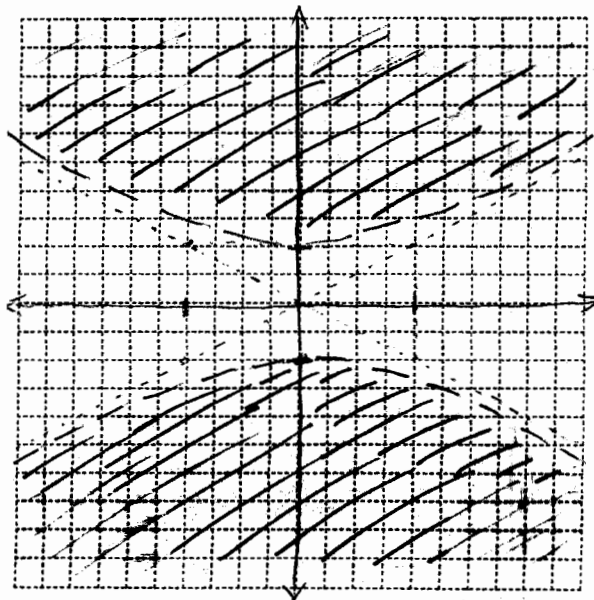
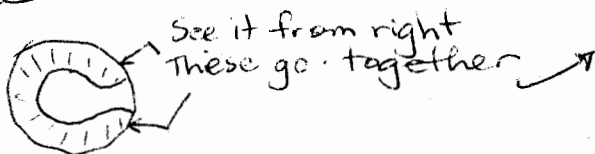
$y^2 - x^2$ opens up and down.
> dotted line.

Step 2: Test center (0,0)

$$\frac{0^2}{4} - \frac{0^2}{16} > 1$$

$0 > 1$ false

The shaded area of a hyperbola is like the fabric on a baseball; it "wraps around"



③ cont $\begin{cases} 4y^2 > x^2 + 16 & (A) \\ y \geq x^2 - 3 & (B) \end{cases}$

But wait! We've only done the hyperbola. We have to add the other graph and determine where the shaded areas overlap.

step 3: Graph $y \geq x^2 - 3$

$y, x^2 \Rightarrow$ parabola up/down
 $a=1$ opens up

$\geq$ solid line

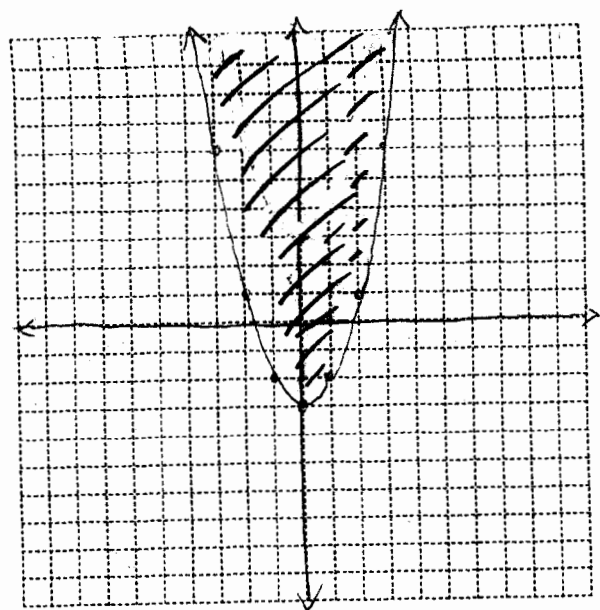
vertex $(0, -3)$

Method 1: test a point not on the line (so do not use the vertex)

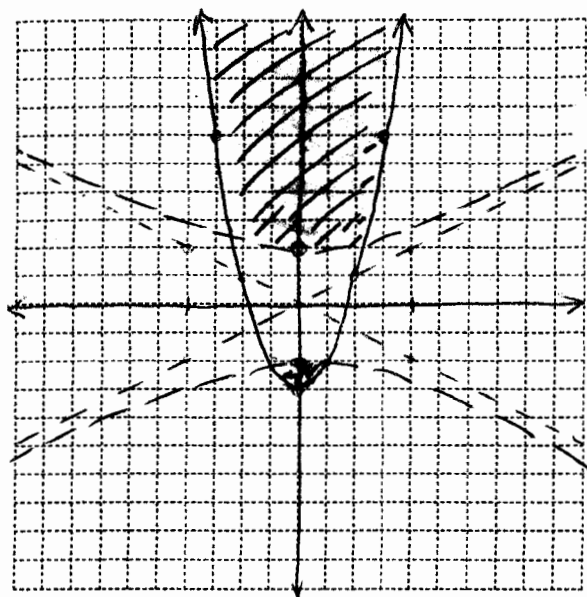
e.g. $(0, 0)$
 $0 \geq 0^2 - 3$
 $0 \geq -3$ true

Method 2: For parabolas

$y \geq$ or $y >$ shade upward from parabola
 $y <$ or $y \leq$ shade downward from parabola
 $x \geq$ or $x >$ shade rightward from parabola
 $x <$ or $x \leq$ shade leftward from parabola



step 4: Shade the overlap

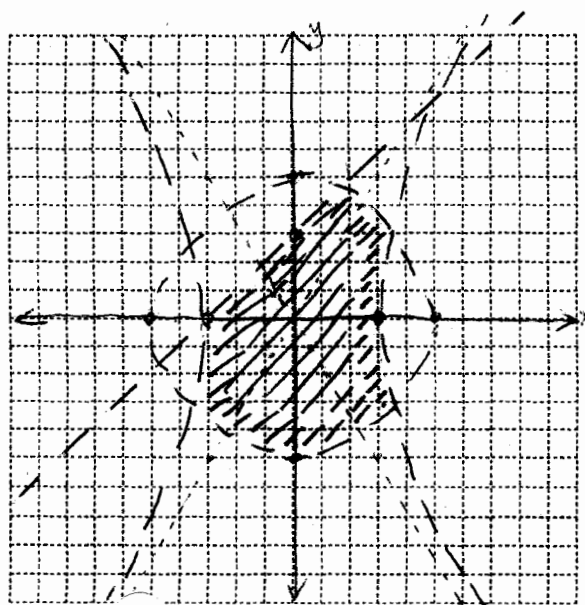


(4) Graph $\begin{cases} x^2 + y^2 < 25 & (A) \\ \frac{x^2}{9} - \frac{y^2}{25} < 1 & (B) \\ y < x + 3 & (C) \end{cases}$

step 1: Graph $x^2 + y^2 < 25$ (A)
 circle center (0,0)
 radius $\sqrt{25} = 5$
 dotted line (<)
 shade inward

step 2: Graph $\frac{x^2}{9} - \frac{y^2}{25} < 1$ (B)
 hyperbola
 center (0,0)
 x-dir 3
 y-dir 5
 opens left & right
 $x^2 - y^2$
 dotted line <
 shade between branches

step 3: Graph $y < x + 3$ (C)
 line
 slope 1
 y-int 3
 dotted line <
 shade downward <

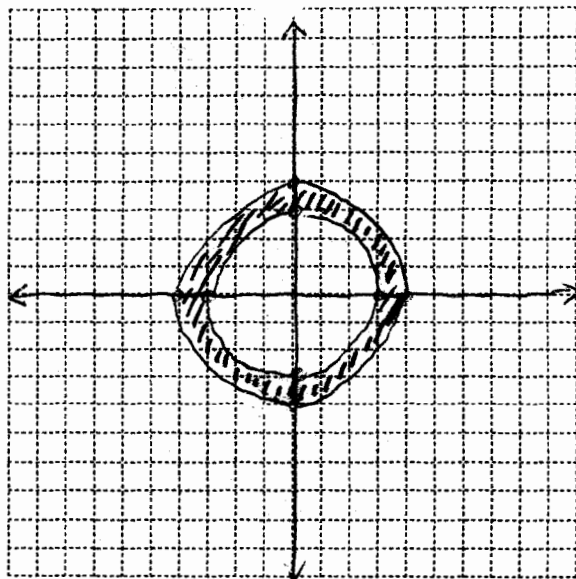


Extras ↓

(5) $\begin{cases} x^2 + y^2 \geq 9 & (A) \\ x^2 + y^2 \leq 16 & (B) \end{cases}$

(A) $x^2 + y^2 \geq 9$
 circle center (0,0)
 radius $\sqrt{9} = 3$
 solid line
 shade outward

(B) $x^2 + y^2 \leq 16$
 circle center (0,0)
 radius $\sqrt{16} = 4$
 solid line
 shade inward



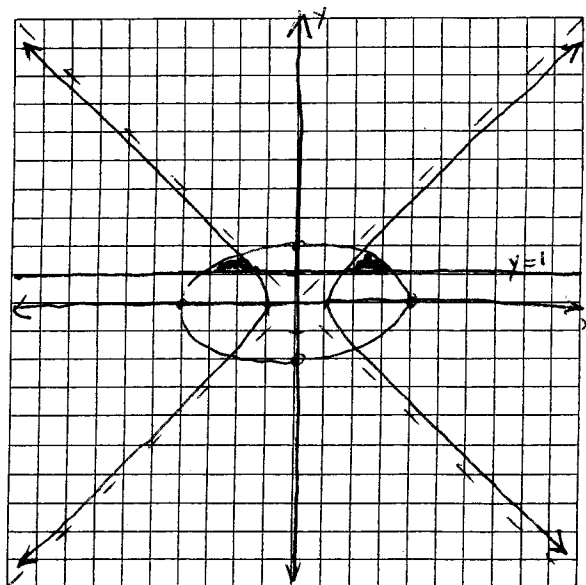
Math 70 10.4

$$\textcircled{6} \begin{cases} x^2 - y^2 \geq 1 & (A) \\ \frac{x^2}{16} + \frac{y^2}{4} \leq 1 & (B) \\ y \geq 1 & (C) \end{cases}$$

(A) $x^2 - y^2 \geq 1$
hyperbola, solid
center (0,0)
 $x^2 - y^2 \rightarrow$ left/right
x-dir 1
y-dir 1
shade outward

(B) $\frac{x^2}{16} + \frac{y^2}{4} \leq 1$
ellipse, solid
center (0,0)
x-dir = 4
y-dir = 2
shade inward

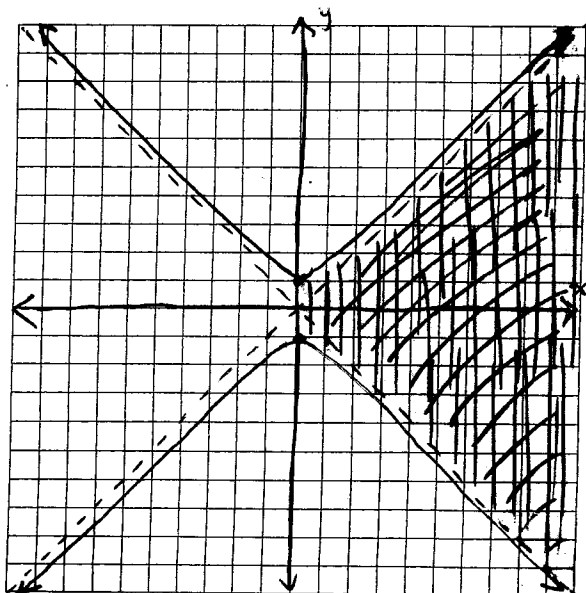
(C) $y \geq 1$
line, solid
horizontal
shade above



$$\textcircled{7} \begin{cases} y^2 - x^2 \leq 1 & (A) \\ x \geq 0 & (B) \end{cases}$$

(A) $y^2 - x^2 \leq 1$
hyperbola, solid
center (0,0)
 $y^2 - x^2 \rightarrow$ up/down
x-dir 1
y-dir 1
shade inward

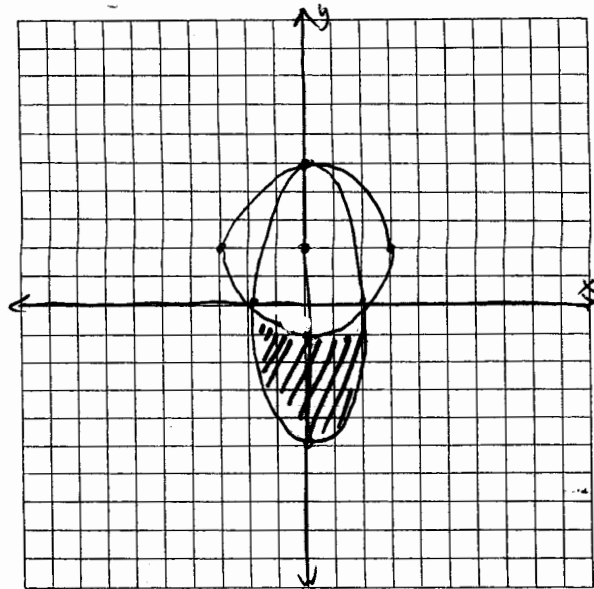
(B) vertical line
 $x \geq 0$ solid
shade right



$$\textcircled{8} \begin{cases} x^2 + (y-2)^2 \geq 9 & (A) \\ \frac{x^2}{4} + \frac{y^2}{25} < 1 & (B) \end{cases}$$

(A) $x^2 + (y-2)^2 \geq 9$
 circle center $(0, 2)$
 solid
 shade out
 radius 3

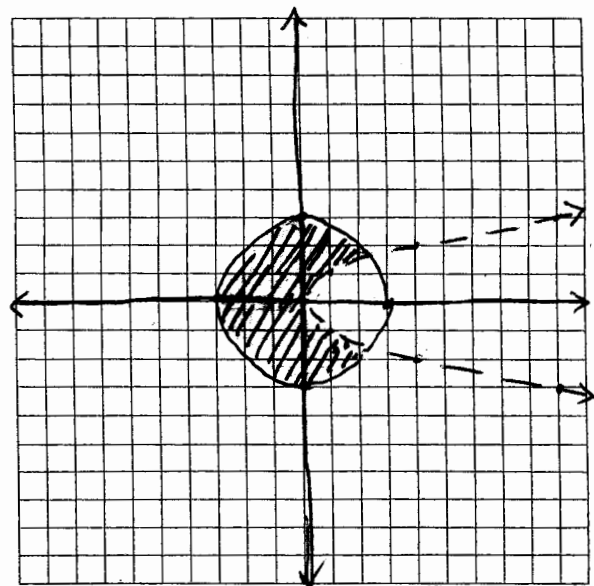
(B) $\frac{x^2}{4} + \frac{y^2}{25} < 1$
 ellipse center $(0, 0)$
 dotted
 shade in
 x-dir 2
 y-dir 5



$$\textcircled{9} \begin{cases} x^2 + y^2 \leq 9 & (A) \\ x < y^2 & (B) \end{cases}$$

(A) $x^2 + y^2 \leq 9$
 circle
 center $(0, 0)$
 radius $\sqrt{9} = 3$
 solid
 shade in

(B) $x < y^2$
 parabola
 vertex $(0, 0)$
 opens right
 basic shape
 dotted line
 shade leftward



Math 70 10.4

$$\textcircled{10} \begin{cases} \frac{x^2}{4} + \frac{y^2}{25} \geq 1 & (A) \\ x < -y^2 - 2y + 6 & (B) \end{cases}$$

$$(A) \frac{x^2}{4} + \frac{y^2}{25} \geq 1$$

ellipse

center (0,0)

x-dir 2

y-dir 5

solid border

shade out

$$(B) x < -y^2 - 2y + 6$$

parabola

$x = y^2$ left/right

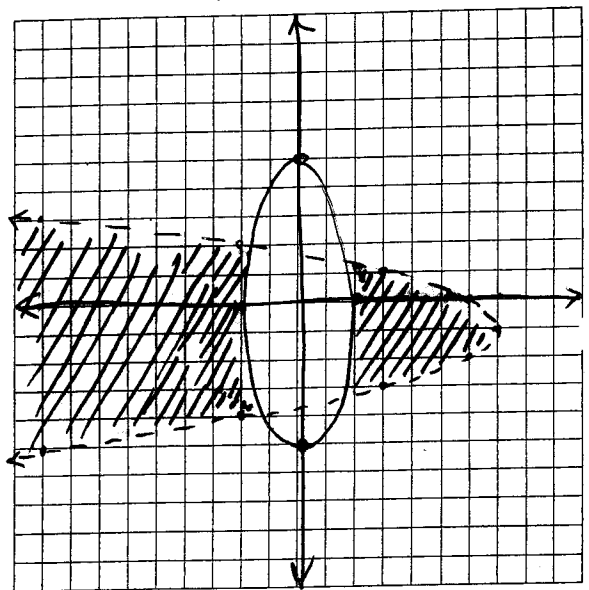
$a = -1$ left

$$y = \frac{-b}{2a} = \frac{-(-2)}{2(-1)} = -1$$

$$x = -(-1)^2 - 2(-1) + 6 = 7$$

dotted border

shade left



Math 70 EXAM #5 Topics List

Math 70-33: Exam #5 Wednesday, 16 May, 1:20 PM – 2:25 PM

Math 70-34: Exam #5 Thursday, 17 May, 1:20 PM – 2:25 PM

Optional Exam 5 Review session / office hours for any Math 70 student in either section:

Friday, May 11, 1-3 PM, room 342.

Martin-Gay		Bittinger
Lecture sections	Lesson number	Lecture sections
8.5	44	11.6
8.6	45	11.7
8.6	46	11.8*
	47	(Previous exam)
dist, midpoint & 10.1	48	10.7 & 13.1-1st
10.1	49	13.1-2nd
10.2	50	13.2
10.2	51	13.3*
10.3	52	13.4
10.4	53	13.5
11.1	54	14.1-1st
11.3	55	14.1-2nd
Review	56	Review
	57	EXAM 5 (c11.6-11.8, c13, c14)

If you are repeating Math 70 or study with students in other sections of Math 70, it may be helpful to know how the other textbook, Martin-Gay, aligns with your textbook, Bittinger.

Math 70 Summary for Exam 5
Chapters 10, 13, 14

10.7 Distance formula

Midpt formula

13.1 Circles: graph from eq'n in standard form
write eq'n from center + radius
write eq'n $ax^2 + bx + cy^2 + dy + e = 0$
in standard form

Parabolas: determine if opening left or right
or up or down

y & x^2 vs x & y^2

$a > 0$ or $a < 0$

vertex formula for left or right -
opening parabolas

using pattern of squares to graph
left or right parabolas

axis of symmetry: graph + eq'n
for left or right parabolas.

Use GC to graph

solve for y

use $\pm$ when taking square roots

use two $y =$ lines in your $y =$ menu

13.2 Ellipses: graph from eq'n in standard form

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

center $(h, k) \rightarrow$ #s in parentheses,
take opposite
coordinate is next to its
variable

x-dir $\sqrt{a^2} = a$ under x^2

y-dir $\sqrt{b^2} = b$ under y^2

13.3 Hyperbolas: graph from eq'n in standard form
asymptotes have slopes $\pm \frac{y\text{dir}}{x\text{dir}}$

opens left & right if x^2 first
 " up & down if y^2 first

13.4 Solving nonlinear systems of equations

* strictly algebra

- elimination method — combine like terms only
- substitution method — FOIL if needed.

if either the x or y coordinate is imaginary,
 no solution.

Visualize shape of graph, anticipate # solns possible.

13.5 Graphing nonlinear inequalities

$\{ < \text{ or } >$ use dashed boundary
 $\{ \leq \text{ or } \geq$ use solid boundary

$\{ y^2 \text{ or } y \leq$ lines or parabolas — shade down
 $\{ y > \text{ or } y \geq$ " " — " up

$\{ x < \text{ or } x \leq$ lines or parabolas — shade left
 $\{ x > \text{ or } x \geq$ " " — " right

$\left\{ \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} \leq 1 \text{ or } < 1 \right.$ ellipses shade inside
 $\left. \geq 1 \text{ or } > 1 \right.$ " " outside

$\left\{ \frac{(\square)^2}{\square} - \frac{(\square)^2}{\square} \leq 1 \text{ or } < 1 \right.$ hyperbolas shade between curves
 $\left. \geq 1 \text{ or } > 1 \right.$ " both "tongues"
 ex: 

14.1 Sequences

Find individual terms (#) given general term a_n

Write general term a_n given individual terms (#)

Series — (sums)

Write a series using sigma or summation notation.